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A Study on Runup of Nonbreaking Double Solitary Waves on Plane Slope

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Abstract

The propagation and runup of the double solitary waves propagating in water of constant depth and climbing along a plane slope are studied experimentally and numerically. Well-designed run-up experiment and numerical simulation of double solitary waves are carried out. Time series of the surface elevation and waterline movement, are obtained. With regard to double solitary waves with same height, the runup amplification coefficient of the latter wave is less than that of the leading one when the relative wave crest distance is reduced to a certain threshold value. The detailed velocity field and energy budget are obtained using the numerical model and discussed in terms of characteristics of velocity field at the runup process of the double solitary waves.

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1. Introduction

It has been a traditional subject to understand the propagation of a solitary wave over a constant depth region and its runup against a sloping beach because of importance of evaluating potential inundation and impacts of tsunami on coastal structures.

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Synolakis (1987) developed an analytical solution for nonbreaking waves on a plane beach based on the nonlinear shallow water equation. Li & Raichlen (2001) proposed a nonlinear solution to the classical shallow water equation by using a hodograph transformation and reported an experimental study on runup process of nonbreaking and breaking solitary wave. Fuhrman and Madsen (2008) proposed the reduced surf similarity parameter for solitary waves, the beach slope divided by the offshore wave height to depth ratio, which provides good coherency with experimental breaking and runup data and analytical nonbreaking runup expressions.

The full nonlinear and highly dispersive Boussinesq equations were used by Zhao *et al.* (2012) to investigate the evolution and run-up of solitary waves and N-waves on plane beaches. They discussed variations of the potential energy and the kinematic energy during the run-up and rundown on a plane beach. Further, Zhao *et al.* (2013) carried out numerical simulation of tsunami waves propagating on the continental shelf with an extremely gentle slope). It is interesting to note that, from the numerical results obtained by the Boussinesq model, the N-shape tsunami waves could evolve into long wave trains, undular bores or solitons near the coastal area for the cases of different initial wave heights. Recently, Chan and Liu (2012) extended the analytical approach of Madsen and Schäffer (2010) to study the runup formulae of a train of solitary waves and concluded that, for the runup of two solitary waves where a wave is followed by one with a larger amplitude, the maximum runup is slightly smaller than that of a single solitary wave. Xuan *et al.* (2013) reported an experimental study of runup double solitary wave on a plane beach. Even though much work has been done about the overtaking or head-on collision of two solitary waves theoretically, particularly at the multi-solitary wave solution of the KdV equation, there are few studies on detailed description of the variation of velocity field and energy of a wave train of multiple soliton-like waves in the process of runup on a plane beach.

This paper presents an investigation on runup of the double solitary waves on a plane beach experimentally and numerically. The runup of the double solitary waves with different wave height on plane beaches are computed and compared with the measured data. Second, the details of the velocity field are obtained through numerical simulation at first. Then, the changes of the potential energy and the kinetic energy of the double solitary waves in the runup process are discussed.

2. Experimental Setup and Numerical Model

2.1. Experimental setup

We carried out experiments on the runup characteristics of two solitary waves in the wave flume of the MOE Key Laboratory of Hydrodynamics at Shanghai Jiao Tong University. The facility consists of a wave flume (65m long \times 1.8m deep \times 0.8m wide), a piston-type wave generation system and a wave elevation measurement system. A piston-type wave generator is installed at the left end of the wave flume and the paddle is moved horizontally in a prescribed trajectory by means of a hydraulic servo-system. At the right end of the wave flume, a slope beach is installed to eliminate the wave reflection. There are total 12 wave gauges along the wave flume, includes 11 gauges in the constant depth region and the remaining one at the slope to record the moving shoreline. The maximum wave runup is recorded by two high-speed cameras with 100Hz record frequency and 1024 \times 1024 resolution.

Goring (1978) developed a solitary wave generation methodology in a wave flume which was modified by Malek-Mohammadi and Testik (2010) considering the non-steady-state characteristics of the piston-type wave generator during the wave generation process into account, is used to implement wave generation in a wave flume. Based on the permanent waveform hypothesis, the velocity of water particle in front of the wave paddle is equal to that of the wave paddle, so the velocity of wave paddle can be expressed as,

$$\bar{u}(\xi, t) = \frac{d\xi}{dt} = \sqrt{g \frac{\eta}{d} \left(d + \frac{\eta}{2} \right) \left(\frac{\eta}{d + \eta} \right)} \quad (1)$$

where t is the elapsed time since the start of the motion, $\bar{u}(\xi, t)$ is the depth-averaged horizontal velocity, d is still

water depth and η is the free surface elevation above the still water level.

Using the fourth-order Runge-Kutta method to solve the ordinary differential equation under specified wave profile, we can obtain the wave paddle trajectory ξ . For the two solitary waves, wave paddle stroke $S=2(H_1/k_1d+H_2/k_2d)$ and its duration $T=3.8(1/k_1c_1+1/k_2c_2)+S/2(1/c_1+1/c_2)$. Here, k_1 and k_2 are the wave number, c_1 and c_2 are the celerity, and H_1 and H_2 the wave height of two successive solitary waves respectively. The initial position of the wavemaker paddle is,

$$\xi(-T/2) = -S/2 \quad (2)$$

The third-order solitary wave solutions $\eta(x_1, t)$ are used as the target wave profile in order to improve the accuracy of solitary wave with large amplitude. Assuming the initial positions of two solitary wave crests are located at $x = X_1$ and $x = X_2$. The wave profile of the two solitary waves when $t = 0$ can be expressed as a linear superposition formulation,

$$\frac{\eta(x, 0)}{d} = \alpha_1 \text{sh}^2 \left[1 - \frac{3}{4} \alpha_1 \text{th}^2 + \alpha_1^2 \left(\frac{5}{8} \text{th}^2 - \frac{101}{80} \text{sh}^2 \text{th}^2 \right) \right] + \alpha_2 \text{SH}^2 \left[1 - \frac{3}{4} \alpha_2 \text{TH}^2 + \alpha_2^2 \left(\frac{5}{8} \text{TH}^2 - \frac{101}{80} \text{SH}^2 \text{TH}^2 \right) \right] \quad (3)$$

where $\alpha_1 = H_1/d$ and $\alpha_2 = H_2/d$ are relative wave heights, sh stands for $\text{sech}[k_1(x-X_1)]$ and th stands for $\tanh[k_1(x-X_1)]$; SH stands for $\text{sech}[k_2(x-X_1)+\varepsilon\sigma]$ and TH stands for $\tanh[k_2(x-X_1)+\varepsilon\sigma]$; k_1 and k_2 are wave numbers; $\varepsilon\sigma$ is the phase shift between these two solitary waves, which reflects the separating distance between crests of two solitary waves. Here, σ is defined as $\sigma = k(cT-S)$, T is the duration of plane motion, S is the stroke of plane motion, which adopted the parameters of the posterior solitary wave. Besides, ε is the multiple of phase σ , we adopted five ε values of incident two solitary waves here, including 0.2, 0.4, 0.6, 0.8 and 1.0.

As shown in Fig. 1, several typical wave profiles of two solitary waves with different amplitude and different initial phase can be found, in which the leading one is a solitary wave with smaller amplitude. The horizontal axis is dimensionless time t/T , T stands for the effective time duration defined by Madsen and Schäffer (2010). The vertical axis is the dimensionless wave amplitude, and $H_0 = \text{Max}(H_1, H_2)$ stands for the maximum among H_1 and H_2 of these two successive solitary waves.

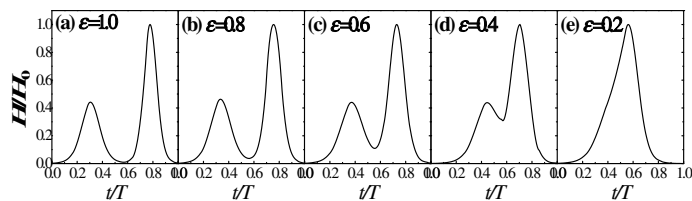


Fig. 1. Two solitary waves with different amplitude.

2.2. Governing equations and numerical aspects

We adopted the Reynolds averaged N-S equations and the RNG $k - \varepsilon$ turbulence model to compute the fluid flows with the free surface. The Volume of Fluid method (VOF) is used to handle with the violent deformation of the free surface. The two-phase mixture model is developed to simulate both air flow and water flow with the complicated free surface motion. The governing equations are formulated in the Cartesian coordinate (x, z) as following

$$\frac{\partial}{\partial x}(uA_x) + \frac{\partial}{\partial z}(wA_z) = 0 \quad (4)$$

$$\begin{aligned}
 \frac{\partial u}{\partial t} + \frac{1}{V_F} \left\{ u A_x \frac{\partial u}{\partial x} + w A_z \frac{\partial u}{\partial z} \right\} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + G_x + f_x \\
 \frac{\partial w}{\partial t} + \frac{1}{V_F} \left\{ u A_x \frac{\partial w}{\partial x} + w A_z \frac{\partial w}{\partial z} \right\} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + G_z + f_z
 \end{aligned}
 \quad (5)$$

in which, t stands for time, x and z are the coordinates, u and w are the horizontal and vertical components of the velocity, p is pressure. V_F is the fluid volume fracture in a computational grid cell in the Volume of Fluid method. A_x and A_z are the area factor of a grid cell. G_x and G_z are the horizontal and vertical components of the body forces, which is taken as $G_x=0$, $G_z=-g$. f_x and f_z represent the viscous terms. Based on the RNG $k-\varepsilon$ turbulence model, the eddy viscosity coefficient is determined as

$$v_T = C_\mu \frac{k_T^2}{\varepsilon_T} \quad (6)$$

where k_T is the turbulent energy, ε_T is the dissipation rate of the turbulent energy, and $C_\mu = 0.085$.

The computational domain is a two dimensional rectangular area, including air and water regions. A moving wavemaker paddle is at the left side and the vertical wall at the right side, as shown in Fig. 2. The motion of the wavemaker paddle is prescribed by the improved Goring's wave generation method as that works for the wave flume.

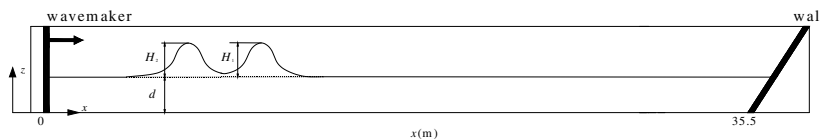


Fig. 2. Computational domain.

2.3. Validation

In order to validate the numerical wave flume, we carried out the numerical tests to check the grid convergence of the numerical model. For the case of water depth $d=0.3\text{m}$ and the initial wave height $H=0.025\text{m}$, three different sizes of the uniform grid are chosen, i.e., $\Delta x = \Delta z = 0.01\text{m}$, $\Delta x = \Delta z = 0.005\text{m}$, $\Delta x = \Delta z = 0.0025\text{m}$. The relative vertical grid size $\Delta z/H$ is equal to 0.4, 0.2 and 0.1 respectively. Fig. 4 presents the comparison among the computed time series of the surface elevation at the waterline on the vertical wall. There is a small phase delay of the maximum runup between the computed value by the coarse grid model of two and half grids from the still water surface and that by other two finer grid model. It is interesting to note that a significant relative difference of the small waves during the rundown process appears for the coarse grid model. The Numerical results show that $\Delta z/H = 0.2$, i.e., five vertical grids in the wave peak region of a solitary wave, can be suggested to obtain an accurate numerical results the propagation and runup of a solitary wave against a vertical wall.

Runup of a solitary on a vertical wall is simulated by the numerical model. The computed maximum runup of a solitary wave on a vertical wall is compared with the measured data, the linear approximation solution proposed by Synolakis (1987) and the third order solution by Su & Mirie (1980). The relationship between the dimensionless runup and the relative wave height of a solitary wave is shown in Fig. 4. The computed runup of a solitary wave agrees well with the measured data and the third order solution of Su & Mirie (1980).

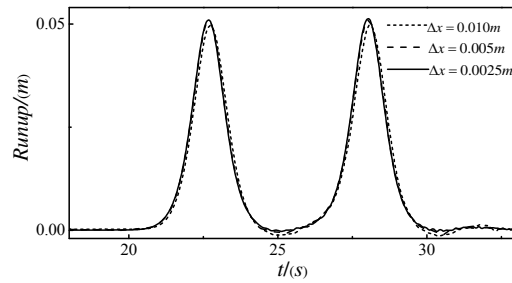


Fig. 3. Time series of the moving waterline computed with different grid sizes.

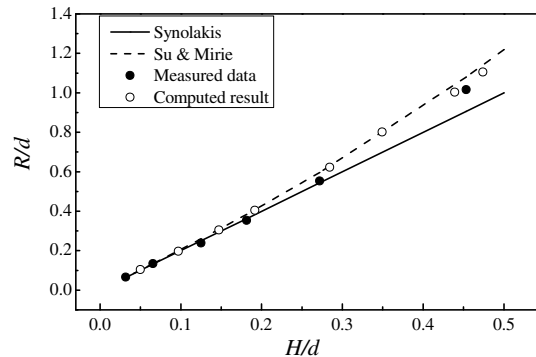


Fig. 4. Comparison among computed, theoretical and measured maximum run-up of solitary wave on vertical wall.

3. Results and Discussion

We define δL as the distance between two peaks of the double solitary waves. The factor χ is defined as $(\delta L/d)/(H_1/H_2)$ which stands for a similarity factor of the two solitary waves of the same amplitude. δL , H_1 and H_2 are measured at location, which is close to the toe of a plane slope. $(R_2/H_2)/(R_1/H_1)$ is ratio of the runup amplification of two waves, indicating the impact of the runup of the leading wave on the runup of successive waves. The numerical results of the maximum runup and the runup amplification coefficients are presented in Table 1.

The runup amplification of the second wave is basically smaller than that of the second wave. The relationship of the new similarity parameter $(R_2/H_2)/(R_1/H_1)$ and χ can be obtained according to the data shown in Table 1. According to the computed results and measured data, an empirical formula is proposed to evaluate the the runup amplification of the double solitary waves of same height on a plane slope, which is independent with the slope angle,

$$\frac{R_2/H_2}{R_1/H_1} = 0.8219\chi^{0.0637} \quad (7)$$

If we look at the interaction of two solitary waves propagating at the same direction along a wave flume, it is expected that, for the case of a solitary wave with smaller amplitude followed by one of larger amplitude at the initial time, the solitary wave with larger amplitude will overtake the solitary wave with smaller amplitude. This is called the overtaking collision of two solitary waves. If the first wave is of larger amplitude at the initial time, these two solitary waves will separate each other because the solitary wave with large amplitude propagates at higher speed. It is interesting to note that the effects of nonlinearity and transient property on the runup amplification of the double solitary waves during the period of runup on a plane slope need more investigation.

The computed velocity fields at typical times during runup and rundown process are plotted in Fig. 5 for the

case of the double solitary waves with same height. The left panel stands for $\varepsilon=0.6$ and the right panel for $\varepsilon=1.0$. The waves propagate from left to right over the flume of water depth 0.3m. When the first solitary wave reaches the maximum runup, the second solitary wave is propagating to the toe of the slope. Then, the head-on collision of the second solitary wave and the reflected solitary wave of the first one appears. When the second solitary wave reaches the maximum runup on the slope, the first solitary wave leaves from the toe of the slope. It is clear that the influence of the first wave on the runup of the second wave is weak if the relative distance between the peaks of the double solitary waves becomes larger.

Table 1. Runup amplification coefficient for the double solitary wave with same wave height

Slope angle	Wave height (m)	ε	$\delta L/d$	H_1/H_2	R_1/H_1	R_2/H_2	χ	$R_2/H_2/R_1/H_1$
30°	0.03	0.6	17.517	0.996	2.437	2.416	17.596	0.991
		0.8	23.283	0.984	2.448	2.431	23.656	0.993
		1.0	28.950	1.005	2.441	2.463	28.804	1.009
	0.05	0.6	14.067	0.994	2.607	2.545	14.153	0.977
		0.8	18.583	0.975	2.626	2.584	19.060	0.984
		1.0	23.083	1.002	2.625	2.641	23.042	1.006
45°	0.03	0.6	17.167	0.997	2.218	2.183	17.224	0.984
		0.8	23.417	1.002	2.241	2.231	23.369	0.995
		1.0	28.684	0.989	2.216	2.228	28.994	1.005
	0.05	0.6	14.067	0.995	2.376	2.290	14.141	0.964
		0.8	18.567	0.975	2.363	2.328	19.037	0.985
		1.0	23.100	1.003	2.378	2.237	23.021	0.999
60°	0.03	0.6	17.317	0.995	2.106	2.069	17.398	0.982
		0.8	23.150	0.984	2.121	2.091	23.535	0.986
		1.0	28.850	1.000	2.123	2.123	28.850	1.000
	0.05	0.6	14.100	0.995	2.217	2.161	14.177	0.975
		0.8	18.517	0.975	2.207	2.171	18.983	0.984
		1.0	23.117	1.003	2.209	2.213	23.045	1.002
75°	0.03	0.6	17.417	0.984	2.063	2.075	17.705	1.006
		0.8	23.250	0.999	2.060	2.077	23.273	1.008
		1.0	29.067	0.992	2.064	2.088	29.305	1.012
	0.05	0.6	14.100	0.994	2.119	2.057	14.182	0.970
		0.8	18.283	0.984	2.095	2.092	18.581	0.999
		1.0	23.050	1.003	2.108	2.137	22.980	1.013

Based on the computed velocity field and the wave profile by the numerical model, the kinetic energy and potential energy of the water in the computational domain can be obtained,

$$E_k = \int_x^{x+X_L} \int_{-h}^{\eta} \frac{1}{2} \rho (u^2 + w^2) dz dx \quad (7)$$

$$E_p = \int_x^{x+X_L} \int_0^{\eta} \rho g z dz dx \quad (8)$$

$$E_n = E_k + E_p \quad (9)$$

in which E_k is the kinetic energy, E_p is the potential energy and $E_n = E_k + E_p$ is the total energy of the double solitary waves during the process of runup and rundown on a slope. X_L is chosen to ensure that the integral area covers the double solitary waves.

Fig.6 depicts the time series of the kinetic energy, potential energy and total energy of the double solitary waves during the period of runup and rundown on a plane slope of the slope angle 30 for the cases of three different phase

delay of initial linear superposition of two waves with same height. The initial heights of the two waves are $H_1=H_2=3\text{cm}$. There are three peaks in the time series of the potential energy and three troughs in that of the kinetic energy. The first and third peaks of the potential energy appear at the time when the first solitary wave and second solitary wave reach the maximum runup respectively. The highest peak of the potential energy appears when the second solitary wave propagating from left to right meets the first reflected solitary wave from the beach. The kinetic energy is basically equal to zero at the head-on collision time. When the initial position of these two solitary waves separate with large distance between two peaks for cases of $\varepsilon=1.0$ and 0.8 , the variation of the energy budge of these two solitary waves in the process of each runup are identical, which means that the influence of the first runup and rundown on the second runup can be ignored. If we look at the bottom panel of Fig. 6 for the case of $\varepsilon=0.6$, where the distance between two peaks is seventeen times as large as water depth, it can be seen that these two runup processes of the double solitary overlap partly and the maximum potential energy for the second runup is smaller than that of the first runup. It confirms the maximum runup of the second wave is smaller than that of the first wave.

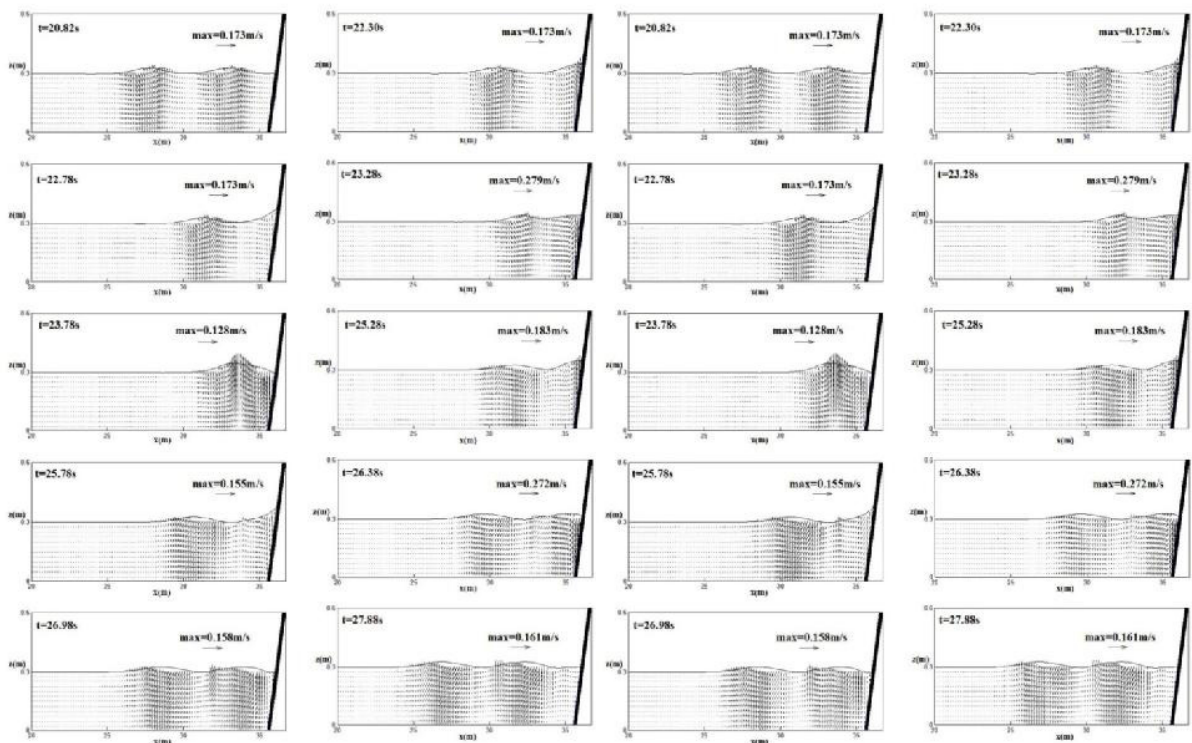


Fig. 5. Velocity fields of the double solitary wave during the process of the run-up on 30° slope (Left panel: $\varepsilon=0.6$; Right panel: $\varepsilon=1.0$).

We also found that the total energy of the double solitary waves decreases during the process of runup and rundown. Basically, the total energy decreases with time which might be induced by the turbulent dissipation because the numerical model is based on the RANS equations and the RNG $k - \epsilon$ turbulence model. It can be expected that, within the time window of runup and rundown process of a solitary wave on a slope, this kind of the energy loss due to turbulent dissipation is small comparing with total energy of the waves. A quantity analysis in the turbulent dissipation will be studied in further work through measuring the velocity field of the double solitary waves by the use of PIV method, particularly at the runup and rundown on a plane slope.

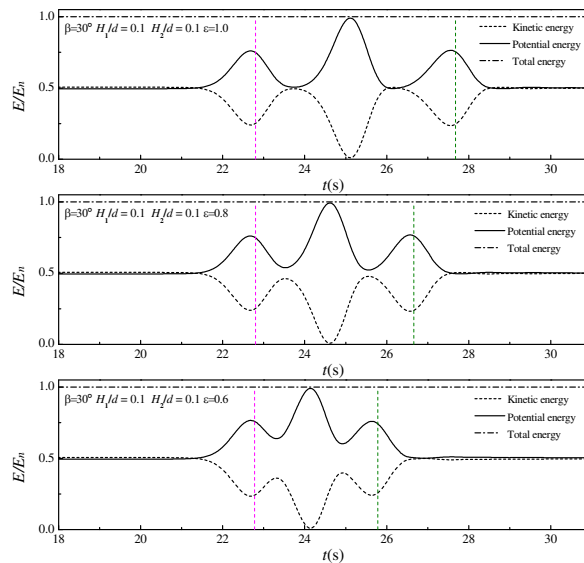


Fig. 6. Time series of potential energy and kinetic energy during run-up and rundown process.

4. Conclusions

Experiments and numerical simulation are carried out to investigate the reflection of the double solitary waves on a plane slope. It is found that the runup of the double solitary waves against a plane slope is weakened by the first wave which results in that the runup amplification coefficient is less than 1.0 as the distance between two wave crests is small. If two wave separate far enough, interaction of two solitary waves during the runup and rundown process on a slope beach is not significant. Three peak pattern of the time series of the potential energy of the double solitary waves reflected by a plane slope, which appear when two single waves reach the maximum runup and the head-on collision of the second single wave and the first reflected one in the front of the wall.

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